

# **Using Manipulates in Algebra**

**Upper Moreland High School Math Department**

# Appendix

## Representing Expressions

In this appendix, you will use tools called “algebra tiles” to represent expressions and equations physically and visually. You will lay a strong foundation for simplifying expressions and solving equations that you will need and build upon repeatedly throughout the course.

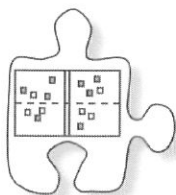
### Guiding Question

Mathematically proficient students make sense of problems and persevere in solving them.

As you work through this chapter, ask yourself:

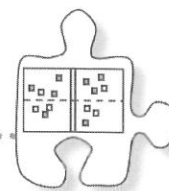
How can algebra tiles help me make sense of and solve problems with variables, expressions, and equations?

### Appendix Outline



This appendix introduces algebra tiles to help you develop the symbolic-manipulation skills of writing and simplifying algebraic expressions. A special focus will be placed on the meaning of “minus” and how to make “zero.” You will also solve for a variable if you know that two expressions are equal.

## A.1.1 What is a variable?

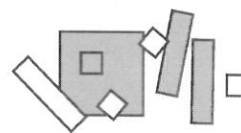


### Exploring Variables and Combining Like Terms

When you worked on problems in Chapter 1, you may recall the dimensions of the hot tub were unknown, the length of time for one of your team members to sign her/his name was unknown, and the output of a function machine was unknown. In many of the problems, some quantity was unknown.

In Algebra and in future mathematics courses, you will continue to work with unknown quantities that can be represented using **variables**. Today you will be introduced to tools called “algebra tiles” that will help you and your team members answer some important questions about algebra, such as “What is a variable?” and “How can we use it?”

A-1. Your teacher will give you and your team a set of algebra tiles to use during this algebra course. As you explore the tiles, discuss the following questions with your team. Be prepared to share your responses with the class. Templates for the tiles are also available at [www.cpm.org/students/resources.htm](http://www.cpm.org/students/resources.htm).



- How many different shapes can you find? What are the names of all of the different shapes?
- How are the shapes different? How are they the same?
- How are the different kinds of shapes related? Which fit together and which do not?

A-2. Draw a picture of each size of tile on your paper. Then complete the activities below.

- The algebra tiles will be referred to by their areas. Since the smallest square has sides 1 unit long, its area is 1 square unit. The name for this tile, then, is “one.” It can also be called a “unit tile.” Can you use the unit tile to find the lengths of the sides of the other algebra tiles? Why or why not?
- Name the other tiles using their areas. Be sure to use what you know about the area of a rectangle and the area of a square.

A-3. JUMBLED PILES

Your teacher will show you a jumbled pile of algebra tiles and will challenge you to name all of them. What is the best description for the collection of tiles? Is your description the best possible?

A-4. Build each collection of tiles represented below. Then name the collection using a simpler algebraic expression, if possible. If it is not possible to simplify the expression, explain why not.

a.  $3x + 5 + x^2 + y + 3x^2 + 2$


b.  $2x^2 + 1 + xy + x^2 + 2xy + 5$

c.  $2 + x^2 + 3x + y^2 + 4y + xy$

d.  $3y + 2 + 2xy + 4x + y^2 + 4y + 1$

A-5. In your Learning Log, using your own words, explain what a variable is. Then sketch a diagram of each type of algebra tile, recording each dimension and an area label. Explain when tiles can and cannot be combined, and include examples to support your statements. Title this entry "Variables" and include today's date.



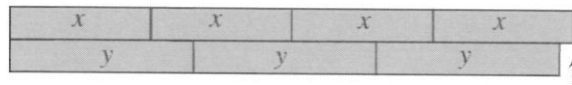


MATH NOTES

## METHODS AND MEANINGS

### Non-Commensurate

Two measurements are called **non-commensurate** if no combination of one measurement can equal a combination of the other. For example, your algebra tiles are called non-commensurate because no combination of unit squares will ever be exactly equal to a combination of  $x$ -tiles (although at times they may appear close in comparison). In the same way, in the example below, no combination of  $x$ -tiles will ever be exactly equal to a combination of  $y$ -tiles.



No matter what number of each size tile, these two piles will never exactly match.

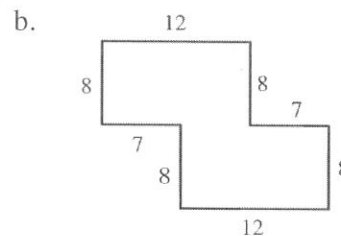
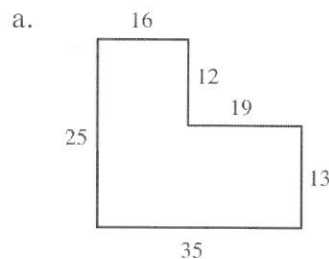




A-6. Suppose you put one of your  $x$ -tiles and two unit tiles with another pile of three  $x$ -tiles and five unit tiles. What is in this new pile? Write it as a sum.

A-7. Suppose one person in your team has two  $x^2$ -tiles, three  $x$ -tiles, and one unit tile on his desk and another person has one  $x^2$ -tile, five  $x$ -tiles, and eight unit tiles on her desk. You decide to put all of the tiles together on one desk. What could you call this new group of tiles? Write it as a sum.

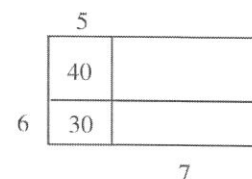
A-8. Copy the following figures onto your paper. Then find the area and perimeter of each shape, assuming that all corners are right angles. Be sure to show all of your work.



A-9. Consider the rectangle at right.

a. Find the perimeter of the large outer rectangle shown at right.

b. Notice that the areas of two of the parts have been labeled inside the rectangle. Find the total area. Remember to show all work leading to your solution.



- A-10. The word “evaluate” has many different meanings. In algebra, when you are asked to **evaluate** an expression for a specified value of the variable(s), you are being asked to find the value of the expression. You do this by replacing a variable with a number and calculating the result. For example, if you are asked to evaluate the expression  $4x - 2$  when  $x = -7$ , you would put  $-7$  in place of the variable and then calculate:  $4 \cdot (-7) - 2 = -30$ .

Evaluate the expressions below for the given values of  $x$  and  $y$ .

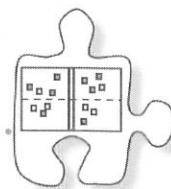
- a.  $\frac{6}{x} + 9$  if  $x = 3$                       b.  $8x - 3 + y$  if  $x = 2$  and  $y = 1$   
c.  $2xy$  if  $x = 5$  and  $y = -3$               d.  $2x^2 - y$  if  $x = 3$  and  $y = 8$

- A-11. Simplify each expression.

- a.  $-\frac{1}{2} + \frac{3}{4}$               b.  $-\frac{1}{3} - \frac{1}{6}$               c.  $-\frac{2}{3} \cdot 12$               d.  $-4 \div -\frac{1}{2}$

## A.1.2 What is the perimeter?

### Simplifying Expressions by Combining Like Terms



While Lesson A.1.1 focused on the *area* of an algebra tile, today’s lesson will focus on the *perimeter*. What is perimeter? How can you find it? By the end of this lesson, you will be able to find the perimeters of complex shapes that are formed with multiple tiles.

You will learn various ways to find perimeter, recognizing that there are different ways to “see” perimeter. When you are finding the perimeter of a complex shape, discovering a convenient shortcut can help you find the perimeter more quickly. As you work through this lesson’s activities, be sure to share any insight into finding perimeter with your teammates and with the whole class.

While working today, ask yourself and your teammates these focus questions:

How did you see it?

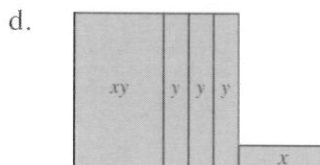
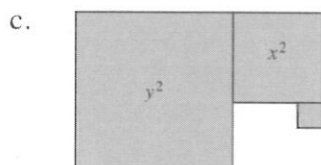
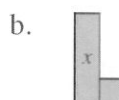
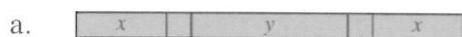
How can you write it?

Is your expression as simplified as possible?

- A-12. Your teacher will provide a set of algebra tiles for you and your team to use today. After you receive the tiles, find one tile of each shape. Then, for each shape, review its name (area). Next, find the *perimeter* of each tile. Decide with your team how to write a simplified expression that represents each perimeter. Be prepared to share the perimeters you find with the class.

- A-13. In parts (a) through (d) below, you will find shapes formed by algebra tiles. Complete the following activities for each shape:

- Use tiles to build the shape.
- Sketch and label the shape on your paper and write an expression that represents the perimeter.
- Simplify the perimeter expression as much as possible.



- A-14. Calculate the perimeter of each shape in problem A-13 if the length of each  $x$ -tile is 3 units and the length of each  $y$ -tile is 8 units. Show all work.

- A-15. EXTENSION

If the perimeter of the shape at right is 32 units, what are possible values for  $x$  and  $y$ ? Is there more than one possible solution for each variable? If so, find another solution. If not, explain how you know there is only one solution.



- A-16. In your Learning Log, create your own shape using three different-shaped algebra tiles. Draw the shape and show how to write a simplified expression for its perimeter. Title this entry “Finding Perimeter and Combining Like Terms” and include today’s date.





## METHODS AND MEANINGS

### Expressions and Terms

A mathematical **expression** is a combination of numbers, variables, and operation symbols. Addition and subtraction separate expressions into parts called **terms**. For example,  $4x^2 - 3x + 6$  is an expression. It has three terms:  $4x^2$ ,  $3x$ , and  $6$ .

A more complex expression is  $2x + 3(5 - 2x) + 8$ , which has three terms:  $2x$ ,  $3(5 - 2x)$ , and  $8$ . However, the term  $3(5 - 2x)$  also has an expression inside the parentheses:  $5 - 2x$ .  $5$  and  $2x$  are terms of this inside expression.

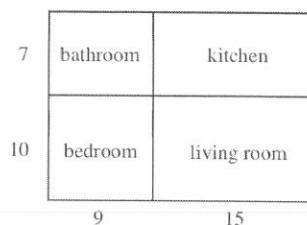


- A-17. Simplify each algebraic expression below, if possible. If it is not possible to simplify the expression, explain why not.
- a.  $3y + 2y + y^2 + 5 + y$                       b.  $3y^2 + 2xy + 1 + 3x + y + 2x^2$
- c.  $3xy + 5x + 2 + 3y + x + 4$                       d.  $4m + 2mn + m^2 + m + 3m^2$
- A-18. Remember that one meaning of the word “evaluate” is to replace a variable with a number and to calculate the result. For example, evaluating the expression  $x^2$  when  $x = -2$  gives the solution  $(-2)^2 = 4$ .

Evaluate the expressions below for the given values.

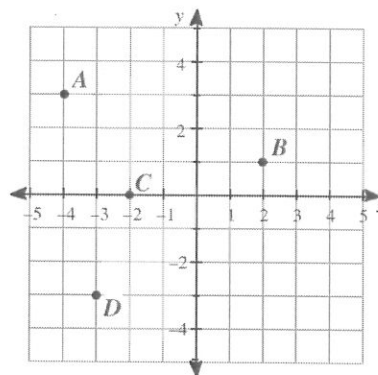
- a.  $-4d + 3$  if  $d = -1$                       b.  $k - m$  if  $k = 4$  and  $m = -10$
- c.  $\frac{t}{w}$  if  $t = 6$  and  $w = -3$                       d.  $x^2 + y^2$  if  $x = 7$  and  $y = 5$

- A-19. The diagram at right is the floor plan of Randy's apartment. Use the diagram to answer the following questions, assuming all measurements are in feet.



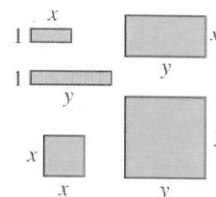
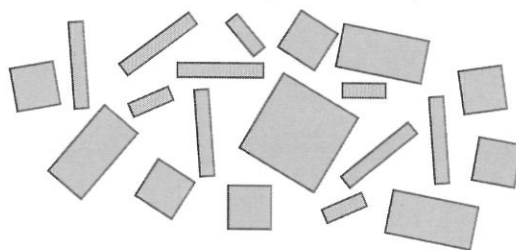
- What are the dimensions (length and width) of Randy's living room?
- Randy's friends are coming to visit him soon. He plans to keep them out of his bedroom. Find the area of each of the other three rooms he will have to clean.
- What is the total area of the rooms he will have to clean?

- A-20. Examine the graph at right. Then complete parts (a) and (b) below.



- In  $(x, y)$  form, name the coordinates of points  $A$ ,  $B$ ,  $C$ , and  $D$ .
- On graph paper, draw a set of axes like the ones shown at right. Then plot points  $E(5, 2)$ ,  $F(-3, -1)$ ,  $G(0, -4)$ , and  $H(2, -3)$ .

- A-21. If algebra tiles have the dimensions shown at right, what would you call the tile collection below? (What is the total area of all of the pieces?) Write the expression algebraically, using  $x$ ,  $x^2$ ,  $y$ ,  $y^2$ , and  $xy$ .



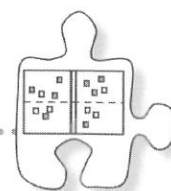
- A-22. Without using a calculator, compute the value of each expression below.

- $-14 + (-31)$
- $-(-8) - (-2)$
- $\frac{-16}{-8}$
- $-11 \cdot 24$
- $\frac{1}{2} - \frac{3}{4}$
- $46 \div (-23)$



# A.1.3 What does “minus” mean?

## Writing Algebraic Expressions



In this section, you will look at algebraic expressions and see how they can be interpreted using an Expression Mat. To achieve this goal, you first need to understand the different meanings of the “minus” symbol, which is found in expressions such as  $5 - 2$ ,  $-x$ , and  $-(-3)$ .

### A-23. LEARNING LOG

What does “ $-$ ” mean? With your team, find as many ways as you can to describe this symbol and discuss how the descriptions differ from each another. Share your ideas with the class and record the different uses in your Learning Log. Title this entry “Meanings of Minus” and include today’s date.



### A-24. USING AN EXPRESSION MAT

Your introduction to algebra tiles in Lessons A.1.1 and A.1.2 involved only positive values. Today you will look at how you can use algebra tiles to represent “minus.” Below are several tiles with their associated values. The shaded tiles are positive, and the un-shaded tiles are negative; the diagram at right will appear throughout the text as a reminder.



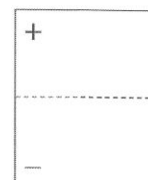
$$\begin{array}{c} \blacksquare \blacksquare \\ \blacksquare \blacksquare \end{array} = 5$$

$$\begin{array}{c} \square \\ \square \end{array} = -3$$

$$\begin{array}{c} \blacksquare x \\ \blacksquare x \end{array} = 3x$$

$$\begin{array}{c} \square y \\ \square y \end{array} = -2y$$

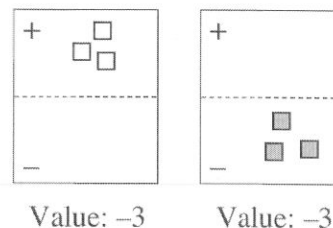
“Minus” can also be represented with a tool called an **Expression Mat**, shown at right. An Expression Mat is an organizing tool that is used to represent expressions. An Expression Mat has a positive region at the top and a negative (or “opposite”) region at the bottom.



*Problem continues on next page. →*

A-24. *Problem continued from previous page.*

The value  $-3$  can be shown in multiple ways, two of which are shown on the Expression Mats at right. Notice that the diagram on the left side uses negative tiles in the “+” region, while the diagram on the right side uses positive tiles in the “-” region.



Use your new knowledge of representing “minus” with an Expression Mat to build the expressions described below.

- Build two different representations of  $-2x$  using an Expression Mat.
- Now build two different representations of  $3x - (-4)$ . How many different ways can you build  $3x - (-4)$ ?

A-25. During your discussion of problem A-24, did you see all of the different ways to represent “minus”? Discuss how you could use an Expression Mat to represent the different meanings discussed in class.

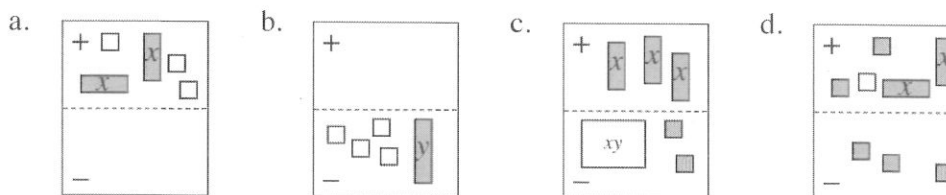
#### A-26. BUILDING EXPRESSIONS

Use an Expression Mat to create each of the following expressions with algebra tiles. Find at least two different representations of each expression. Sketch each representation on your paper and be prepared to share your different representations with the class.

- $-3x + 4$
- $-(y - 2)$
- $-y - 3$
- $5x - (3 - 2x)$

A-27. In problem A-26, you represented algebraic expressions with algebra tiles. In this problem, you will need to reverse your thinking to write an expression from a diagram of algebra tiles.

Working with a partner, write algebraic expressions for each representation below. Start by building each problem with your algebra tiles.



- A-28. Patti, Emilie, and Carla are debating the answer to part (d) of problem A-27. Patti wrote  $2 - 1 + 2x - 3$ . Carla thinks that the answer is  $2x + 2 - 4$ . Emilie is convinced that the answer is  $2x - 2$ . Discuss with your team how each person might have arrived at her answer. Who do you think is correct? When you decide, write an explanation on your paper and justify your answer.



- A-29. Reflect about what you have learned from today's lesson as you answer the following question in your Learning Log. Title this entry "Representing Expressions on an Expression Mat" and include today's date.



Using an Expression Mat, find two different ways to represent  $x - 1 - (2x - 3)$ . Sketch the two different representations and write a few sentences to describe the differences in the ways you built each representation.





## METHODS AND MEANINGS

### Order of Operations

Mathematicians have agreed on an **order of operations** for simplifying expressions.

Original expression:

$$(10 - 3 \cdot 2) \cdot 2^2 - \frac{13 - 3^2}{2} + 6$$

Circle expressions that are grouped within parentheses or by a fraction bar:

$$(10 - 3 \cdot 2) \cdot 2^2 - \frac{13 - 3^2}{2} + 6$$

Simplify *within* circled terms using the order of operations:

- Evaluate exponents.

$$(10 - 3 \cdot 2) \cdot 2^2 - \frac{13 - 3 \cdot 3}{2} + 6$$

- Multiply and divide from left to right.

$$(10 - 6) \cdot 2^2 - \frac{13 - 9}{2} + 6$$

- Combine terms by adding and subtracting from left to right.

$$(4) \cdot 2^2 - \frac{4}{2} + 6$$

Circle the remaining terms:

$$4 \cdot 2^2 - \frac{4}{2} + 6$$

Simplify *within* circled terms using the order of operations as above.

$$4 \cdot 2 \cdot 2 - \frac{4}{2} + 6$$

$$16 - 2 + 6$$

$$20$$



A-30. Copy and simplify the following expressions by combining like terms. Using or drawing sketches of algebra tiles may be helpful.

a.  $2x + 3x + 3 + 4x^2 + 10 + x$

b.  $4x + 4y^2 + y^2 + 9 + 10 + x + 3x$

c.  $2x^2 + 30 + 3x^2 + 4x^2 + 14 + x$

d.  $20 + 5xy + 4y^2 + 10 + y^2 + xy$

- A-31. Plot the points  $A(5, 3)$ ,  $B(-4, 3)$ ,  $C(-4, -6)$ , and  $D(5, -6)$  on a set of axes. Use a ruler to connect them in order, including  $D$  back to  $A$ , to form a **quadrilateral** (a shape with four sides).
- What kind of quadrilateral was formed?
  - How long is each side of the quadrilateral?
  - What is the area of the quadrilateral?
  - What is the perimeter of the quadrilateral?

- A-32. Write an equation to solve the following problem. Write your answer with a complete sentence.

Susan is buying three different colors of tiles for her kitchen floor. She is buying 107 red tiles, and three times as many navy-blue tiles as beige tiles. If Susan buys 435 tiles altogether, how many tiles of each color does she buy?



Let  $b$  = the number of beige tiles. Your equation should start with " $435 =$ " and use  $b$  as the only variable.

- A-33. Without using a calculator, compute the value of each expression below.



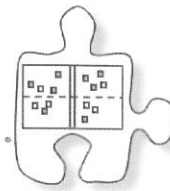
- |                  |                |                      |
|------------------|----------------|----------------------|
| a. $-3 + 6$      | b. $(-2)(-9)$  |                      |
| c. $-4 - 9 - 11$ | d. $-12 - 18$  | e. $\frac{3}{4}(-8)$ |
| f. $(-2)(-2)(2)$ | g. $7 + (-19)$ | h. $-15 \div 15$     |

- A-34. Solve each proportion.

- |                                 |                                |                                  |                      |
|---------------------------------|--------------------------------|----------------------------------|----------------------|
| a. $\frac{6}{8} = \frac{30}{m}$ | b. $\frac{3}{5} = \frac{x}{9}$ | c. $\frac{20}{30} = \frac{y}{9}$ | d. $\frac{x}{3} = 9$ |
|---------------------------------|--------------------------------|----------------------------------|----------------------|

## A.1.4 What makes zero?

### Using Zero to Simplify Algebraic Expressions



Today you will continue your work with rewriting algebraic expressions. As you work with your team, ask yourself and your teammates these focus questions:

How did you see it?

How can you write it?

Is your expression as simplified as possible?

#### A-35. LIKELY STORY!

Imagine the following situations:

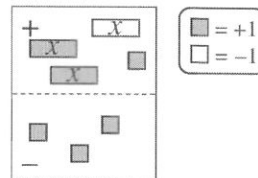
- Ramona babysits her neighbor's baby and stuffs the \$15 she earns into her purse. When she gets home, the \$15 is nowhere to be found. It must have fallen out of her purse.
- The Burton Pumas football team completes a pass and gains 12 yards. But on the very next play, the quarterback holds onto the ball too long and gets sacked, losing 12 yards.
- You are at the beach. You dig a hole in the sand and place the sand you remove in a pile next to your hole. Someone comes along and pushes the pile back into the hole.



What do these situations have in common? Can you represent each of them using math symbols and numbers? How?

- A-36. How can you represent zero with tiles on an Expression Mat? With your team, try to find at least two different ways to do this (and more if you can). Be ready to share your ideas with the class.

- A-37. Gretchen used seven algebra tiles to build the expression shown at right.



- a. Build this collection of tiles on your own Expression Mat and write its value.
- b. Represent this same value three different ways, each time using a *different number* of tiles. Be ready to share your representations with the class.

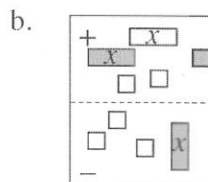
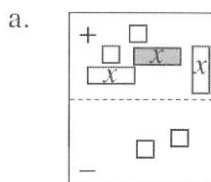
- A-38. Build each expression below so that your representation does not match those of your teammates. Once your team is convinced that together you have found four different, valid representations, sketch your representation on your paper and be ready to share your answer with the class.

a.  $-3x + 5 + y$

b.  $-(-2y + 1)$

c.  $2x - (x - 4)$

- A-39. Write the algebraic expression shown on each Expression Mat below. Build the model and then simplify the expression by removing as many tiles as you can *without changing the value of the expression*. Finally, write the simplified algebraic expression.



- A-40. Simplify each of the following expressions by building it on your Expression Mat and removing zeros. Your teacher will give you instructions about how to represent your work on your paper.

a.  $3x - (2x + 4)$

b.  $7 - (4y - 3) + 2y - 4$

- A-41. In your Learning Log, describe the different ways you can represent zero using your Expression Mat. Include an example and be sure to draw the tiles. Title this entry "Using Zeros to Simplify" and include today's date.





## METHODS AND MEANINGS

### Evaluating Expressions and Equations

The word **evaluate** indicates that the value of an expression should be calculated when a variable is replaced by a numerical value.

For example, when you evaluate the expression  $x^2 + 4x - 3$  for  $x = 5$ , the result is:

$$\begin{aligned}(5)^2 + 4(5) - 3 \\ 25 + 20 - 3 \\ 42\end{aligned}$$

When you evaluate the equation  $y = 3x^2 - 5x + 2$  for  $x = 4$ , the result is:

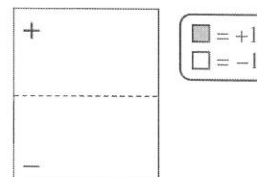
$$\begin{aligned}y &= 3(4)^2 - 5(4) + 2 \\ y &= 3(16) - 5(4) + 2 \\ y &= 48 - 20 + 2 \\ y &= 30\end{aligned}$$

In each case remember to follow the Order of Operations.



- A-42. Bob, Kris, Janelle, and Pat are in a study team. Bob, Kris, and Janelle have algebra tiles on their desks. Bob has two  $x^2$ -tiles, four  $x$ -tiles, and seven unit tiles; Kris has one  $x^2$ -tile and five unit tiles; and Janelle has ten  $x$ -tiles and three unit tiles. Pat's desk is empty. The team decides to put all of the tiles from the three desks onto Pat's desk. Write an algebraic expression for the new collection of tiles on Pat's desk.

- A-43. Can zero be represented by any number of tiles? Using only the unit tiles (only the 1 and  $-1$  tiles), determine if you can represent zero on an Expression Mat with the number of tiles below. If you can, draw an Expression Mat demonstrating that it is possible. If it is not possible, explain why not.



a. 2 tiles

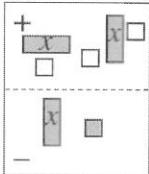
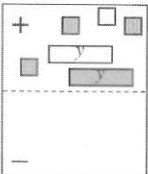
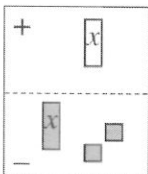
b. 6 tiles


c. 3 tiles

A-44. Read the Math Notes box for this lesson. Then evaluate each equation below.

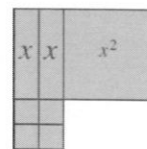
- For  $y = 2 + 3x$  when  $x = 4$ , what does  $y$  equal?
- For  $a = 4 - 5c$  when  $c = -\frac{1}{2}$ , what does  $a$  equal?
- For  $n = 3d^2 - 1$  when  $d = -5$ , what does  $n$  equal?
- For  $v = -4(r - 2)$  when  $r = -1$ , what does  $v$  equal?
- For  $3 + k = t$  when  $t = 14$ , what does  $k$  equal?

A-45. Write and simplify the algebraic expression shown in each Expression Mat below.

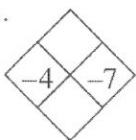
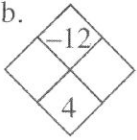
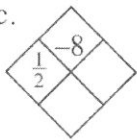
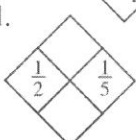
a.  b.  c. 

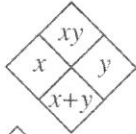


A-46. Write an expression that represents the perimeter of the shape built with algebra tiles at right.



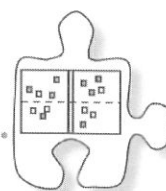
A-47. Copy and complete each of the Diamond Problems below. The pattern used in the Diamond Problems is shown at right.

a.  b.  c.  d. 



## A.1.5 How can I simplify the expression?

### Using Algebra Tiles to Simplify Algebraic Expressions



Which is greater: 58 or 62? That question might seem easy to answer, because the numbers are ready to be compared. However, if you are asked which is greater,  $2x + 8 - x - 3$  or  $6 + x + 1$ , the answer is not so obvious! In this lesson, you and your teammates will investigate how to compare two algebraic expressions and decide which one is greater.

A-48. For each expression below, use an Expression Mat to build the expression. Then find a different way to represent the same expression using tiles.

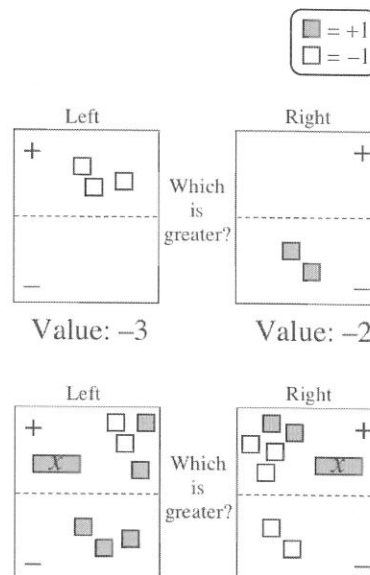
a.  $7x - 3$

b.  $-(-2x + 6) + 3x$

A-49. COMPARING EXPRESSIONS

Two expressions can be represented at the same time using an **Expression Comparison Mat**. The Expression Comparison Mat puts two Expression Mats side-by-side so you can compare them and see which is greater. For example, in the picture at right, the expression on the left represents  $-3$ , while the expression on the right represents  $-2$ . Since  $-2 > -3$ , the expression on the right is greater.

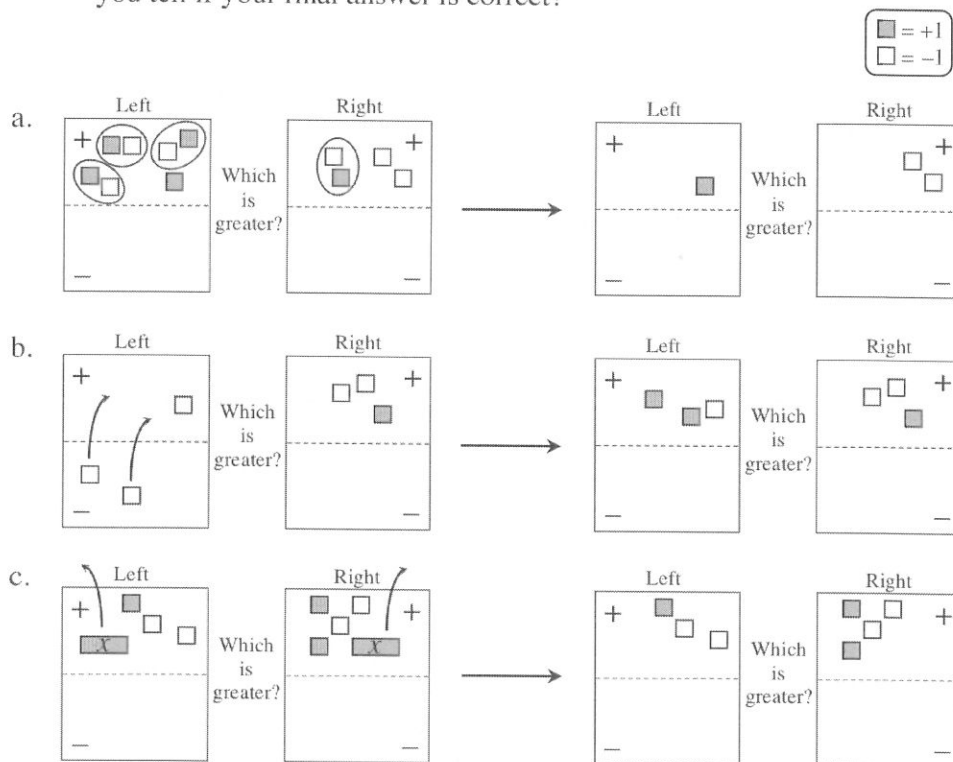
Build the Expression Comparison Mat shown at right. Write an expression representing each side of the Expression Mat.



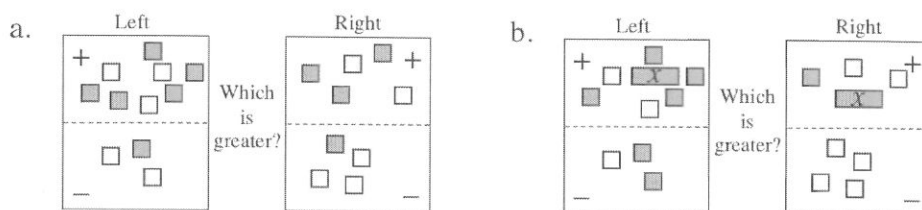
- Can you simplify each of the expressions so that fewer tiles are used? Develop a method to simplify both sides of the Expression Comparison Mats. Why does it work? Be prepared to justify your method to the class.
- Which side of the Expression Comparison Mat do you think is greater (has the largest value)? Agree on an answer as a team. Make sure each person in your team is ready to justify your conclusion to the class.

A-50. As Karl simplified some algebraic expressions, he recorded his work on the diagrams below.

- Explain in writing what he did to each Expression Comparison Mat on the left to get the Expression Comparison Mat on the right.
- If necessary, simplify further to determine which Expression Mat is greater. How can you tell if your final answer is correct?




A-51. Use Karl's "legal" simplification moves to determine which side of each Expression Comparison Mat below is greater. Record each of your "legal" moves on the Lesson A.1.5A Resource Page by drawing on it the way Karl did in problem A-50. After each expression is simplified, state which side is greater (has the largest value). Be prepared to share your process and reasoning with the class.





- A-52. In your Learning Log, explain each of the types of “legal” moves that you can use to simplify and compare expressions. For each type of “legal” move, sketch an example. Title this entry “Legal Moves for Simplifying and Comparing Expressions” and include today’s date.



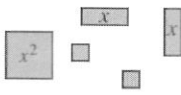


MATH NOTES

## METHODS AND MEANINGS

### Combining Like Terms

Combining tiles that have the same area to write a simpler expression is called **combining like terms**. See the example shown at right.



$x^2 + 2x + 2$

When you are not working with actual tiles, it can help to picture the tiles in your mind. You can use these images to combine the terms that are the same in expressions and equations. Here are two examples:

Example 1:  $2x^2 + xy + y^2 + x + 3 + x^2 + 3xy + 2 \Rightarrow 3x^2 + 4xy + y^2 + x + 5$

Example 2:  $3x^2 - 2x + 7 - 5x^2 + 3x - 2 \Rightarrow -2x^2 + x + 5$

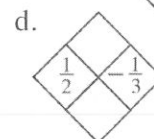
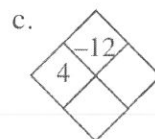
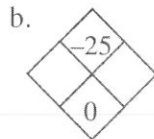
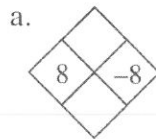
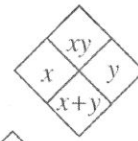
Remember that addition and subtraction separate expressions into **terms**.



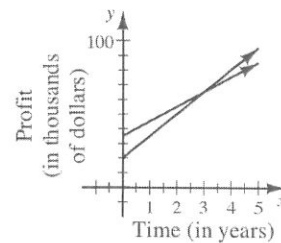
- A-53. Simplify the following expressions by combining like terms, if possible.

- |                                |  |
|--------------------------------|--|
| a. $x + x - 3 + 4x^2 + 2x - x$ | b. $8x^2 + 3x - 13x^2 + 10x^2 - 25x - x$ |
| c. $4x + 3y$                   | d. $20 + 3xy - 3 + 4y^2 + 10 - 2y^2$     |

- A-54. Copy and complete each of the Diamond Problems below.  
The pattern used in the Diamond Problems is shown at right.

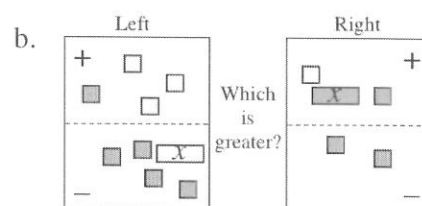
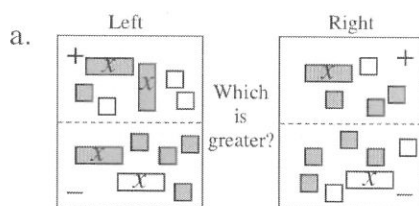


- A-55. The two lines at right represent the growing profits of Companies A and B.



- Sketch this graph on your paper. If Company A started out with more profit than Company B, determine which line represents A and which represents B. Label the lines appropriately.
- In how many years will both companies have the same profit?
- Approximately what will that profit be?
- Which company's profits are growing more quickly? How can you tell?

- A-56. Use legal simplification moves to determine which side of the Expressions Comparison Mat is greater.



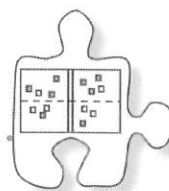
- A-57. Evaluate each expression to find  $y$ .

- $y = 2 + 4.3x$  when  $x = -6$
- $y = (x - 3)^2$  when  $x = 9$
- $y = x - 2$  when  $x = 3.5$
- $y = 5x - 4$  when  $x = -2$

- A-58. Sam read 75 pages of a new mystery novel in 2 hours. If the book contains 350 pages and he always reads at the same rate, how long will it take him to read the entire novel?

# A.1.6 Which is greater?

## Using Algebra Tiles to Compare Expressions



Can you always tell if one algebraic expression is greater than another? In this lesson, you will compare the values of two expressions, practicing the different simplification strategies you have learned in this appendix so far.

### A-59. WHICH IS GREATER?

Write an algebraic expression for each side of the Expression Comparison Mats given below. Use the “legal” simplification moves you worked with in Lesson A.1.5 to determine which expression on the Expression Comparison Mat is greater.



a. **Left** **Right**

Which is greater?

b. **Left** **Right**

Which is greater?

c. **Left** **Right**

Which is greater?

d. **Left** **Right**

Which is greater?

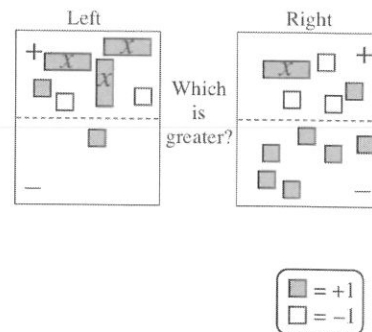
e. **Left** **Right**

Which is greater?

f. **Left** **Right**

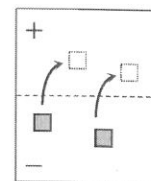
Which is greater?

- Simplify the expressions using the “legal” moves that you developed in Lesson A.1.5.
- Can you tell which expression is greater? Explain your answer in a few sentences on your paper. Be prepared to share your conclusion with the class.

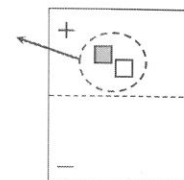


## Simplifying an Expression

- Flip tiles and move them from the negative region to the positive region (that is, find the opposite). For example, the two unit tiles in the “-” region can be flipped and placed in the “+” region.

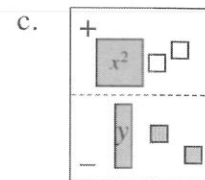
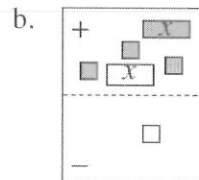
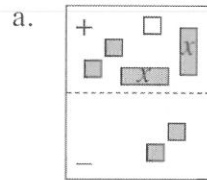


- Remove an equal number of opposite tiles that are in the same region. For example, the positive and negative tile to the right can be removed.

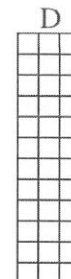
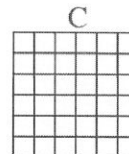
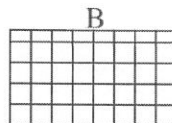
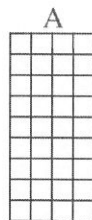




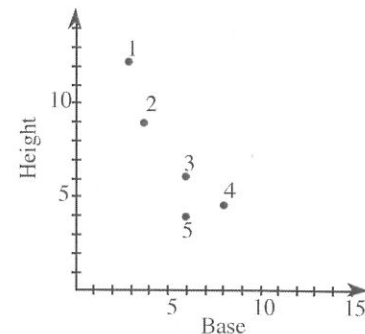
- A-61. Find a simplified algebraic expression for each Expression Mat below.



- A-62. Cairo wants to create a graph representing the heights and bases of all rectangles that have an area of 36 square units. He started by drawing the rectangles A, B, C, and D below. Examine the dimensions (length and width) of each rectangle.



- Copy the graph at right onto graph paper. Then match the letter of each rectangle above with a point on the graph. Which point is not matched?
- What are the base, height, and area for the unmatched point?
- Why should the unmatched point not be on Cairo's graph?
- Find the dimensions of three more rectangles that have areas of 36 square units. At least one of your examples should have dimensions that are not integers. Place a new point on the graph for each new rectangle you find.
- Connect all of the points representing an area of 36 square units. Describe the resulting graph.



A-63. Without using a calculator, compute the value of each expression below.

a.  $7 - 2 \cdot (-5)$

b.  $6 + 3(7 - 3 \cdot 2)^2$

c.  $5 \cdot (-3)^2$

d.  $35 \div (16 - 3^2) \cdot 2$

e.  $-3 \cdot 4 + 5 \cdot (-2)$

f.  $7 - 6(10 - 4 \cdot 2) \div 4$



A-64. One of Teddy's jobs at home is to pump gas for his family's sedan and truck. When he fills the truck up with 12 gallons of gas, he notices that it costs him \$39.48.

- How much does one gallon of gas cost? Explain how you found your answer.
- How much will it cost him to fill up the sedan if it needs 15 gallons of gas? Show your work.
- When Teddy filled up the tank on his moped, it cost \$13.16. How much gas did his moped need? Explain how you know.

A-65. Draw a circle on your paper and lightly shade in three-fourths of the circle.

- Divide the entire circle into eight equal parts. How many parts are shaded?
- Using fractions, write and solve a related division problem.

A-66. If  $f(x) = 2x + 3$ , evaluate each expression below.

a.  $f(-5)$

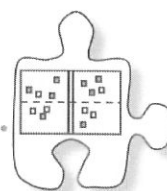
b.  $f(-\frac{1}{2})$

c.  $f(4)$

d.  $f(-2)$

## A.1.7 How can I write it?

### Simplifying and Recording Work



Today you will continue to compare expressions as you strengthen your simplification strategies. At the same time, you will work with your class to find ways to record your work so that another student can follow your strategies.

A-67. Use algebra tiles to build the expressions below on an Expression Comparison Mat. Use “legal” simplification moves to determine which expression is greater, if possible. If it is not possible to tell which expression is greater, explain why.

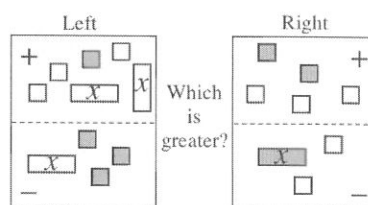
- Which is greater:  $3x - (2 - x) + 1$  or  $-5 + 4x + 4$ ?
- Which is greater:  $2x^2 - 2x + 6 - (-3x)$  or  $-(3 - 2x^2) + 5 + 2x$ ?
- Which is greater:  $-1 + 6y - 2 + 4x - 2y$  or  $x + 5y - (-2 + y) + 3x - 6$ ?

### A-68. RECORDING YOUR WORK

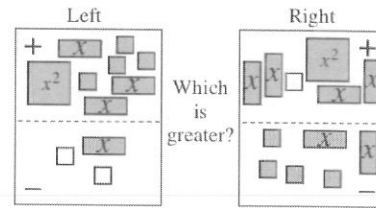
Although using algebra tiles can make some situations easier because you can “see” and “touch” the math, it can be difficult to remember what you did to solve a problem unless you take good notes.



Use the simplification strategies you have learned to determine which expression on the Expression Comparison Mat at right is greater. Record each step as instructed by your teacher. Also record the simplified expression that remains after each move. This will be a written record of how you solved this problem. Discuss with your team what the best way is to record your moves.



- A-69. While Athena was comparing the expressions shown at right, she was called out of the classroom. When her teammates needed help, they looked at her paper and saw the work shown below. Unfortunately, she had forgotten to explain her simplification steps.

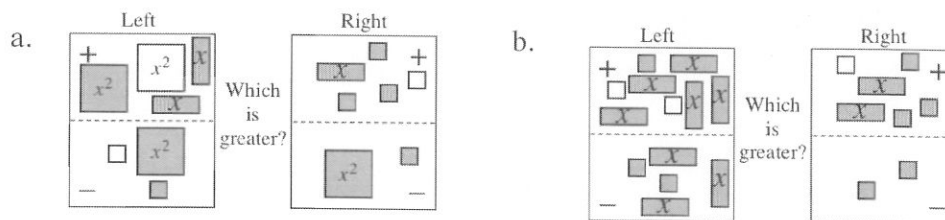


Can you help them figure out what Athena did to get each new set of expressions?



Left Expression	Right Expression	Explanation
$3x + 4 - x - (-2) + x^2$	$-1 + x^2 + 4x - (4 + 2x)$	Original expressions
$3x + 4 - x - (-2)$	$-1 + 4x - (4 + 2x)$	
$3x + 4 - x + 2$	$-1 + 4x - 4 - 2x$	
$2x + 6$	$2x - 5$	
$6$	$-5$	
Because $6 > -5$ , the left side is greater.		

- A-70. For each pair of expressions below, determine which is greater. Carefully record your steps as you go. If you cannot tell which expression is greater, write, “Not enough information.” Make sure that you record the result after each type of simplification. For example, if you flip all of the tiles from the “-” region to the “+” region, record the resulting expression and use words or symbols to indicate what you did. Be ready to share your work with the class.



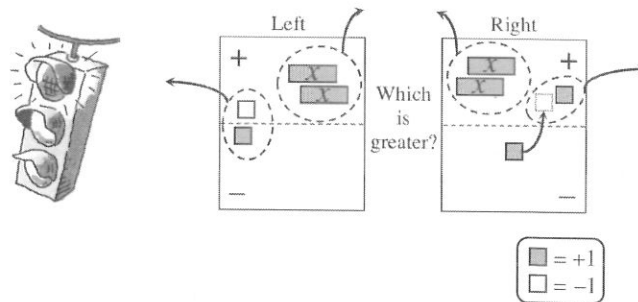
- c. Which is greater:  $5 - (2y - 4) - 2$  or  $-y - (1 + y) + 4$ ?
- d. Which is greater:  $3xy + 9 - 4x - 7 + x$  or  $-2x + 3xy - (x - 2)$ ?



- A-71. Solve this problem by writing an equation. Write your answer with a complete sentence. Let  $x$  = the number of adults. Your equation should start with “1220 =” and use  $x$  as the only variable.

The number of students attending the fall play was 150 more than the number of adults attending. A total of 1220 people attended the play. How many students attended the play?

- A-72. Sylvia simplified the expressions on the Expression Comparison Mat shown at right. Some of her work is shown. Are all of her moves “legal”? Explain.



- A-73. Examine the tile pattern at right.

- a. On graph paper, draw Figures 4 and 5.

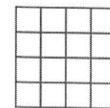


Figure 1

Figure 2

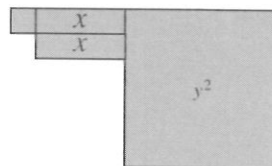
Figure 3

- b. What would Figure 10 look like?  
How many tiles would it have on each side?  
What about Figure 100?

- c. Cami has a different tile pattern. She decided to represent the number of tiles of her pattern in a table, as shown below. Can you use the table to predict how many tiles would be in Figure 5 of her tile pattern? How many tiles would Figure 8 have? Explain how you know.

Figure Number	1	2	3	4
Number of Tiles	5	9	13	17

A-74. Examine the shape made with algebra tiles at right.



- a. Write an expression that represents the perimeter of the shape. Then evaluate your expression for  $x = 6$  and  $y = 10$  units.
- b. Write an expression that represents the area of the shape. What is the area if  $x = 6$  and  $y = 10$  units?

A-75. CALCULATOR CHECK

Use your scientific calculator to compute the value of each expression in the left-hand column below. Match each result to an answer in the right-hand column.

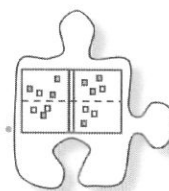


- |                            |           |
|----------------------------|-----------|
| a. $-3 + 16 - (-5)$        | 1. $-16$  |
| b. $(3 - 5)(6 + 2)$        | 2. $327$  |
| c. $17(-23) + 2$           | 3. $0.5$  |
| d. $5 - (3 - 17)(-2 + 25)$ | 4. $18$   |
| e. $(-4)(-2.25)(-10)$      | 5. $-90$  |
| f. $-1.5 - 2.25 - (-4.5)$  | 6. $0.75$ |
| g. $\frac{4-5}{-2}$        | 7. $-389$ |

A-76. Alicia used 4 gallons of gasoline to travel 90 miles. At the same rate, how far can she travel on a full tank that holds 18 gallons?

# A.1.8 What if both sides are equal?

Using Algebra Tiles to Solve for  $x$



Can you always tell whether one algebraic expression is greater than another? In this section, you will continue to practice the different simplification strategies you have learned so far to compare two expressions and see which one is greater. However, sometimes you do not have enough information about the expressions. When both sides of an equation are equal, you can learn even more about  $x$ . As you work today, focus on these questions:

How can you simplify?

How can you get  $x$  alone?

Is there more than one way to simplify?

Is there always a solution?

## A-77. WHICH IS GREATER?

Build each expression represented below with the tiles provided by your teacher. Use “legal” simplification moves to determine which expression is greater, if possible. If it is not possible to determine which expression is greater, explain why it is impossible. Be sure to record your work on your paper.

a.

Left		Right	
<div style="border: 1px solid black; padding: 5px; width: 100px;"> <math>+</math>  </div>	Which is greater?	<div style="border: 1px solid black; padding: 5px; width: 100px;"> <math>+</math>  </div>	<div style="border: 1px solid black; padding: 5px; width: 50px;"> </div>

b. Which is greater:  $x + 1 - (1 - 2x)$  or  $3 + x - 1 - (x - 4)$ ?

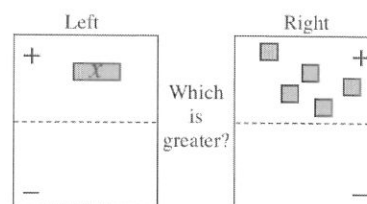
A-78. WHAT IF BOTH SIDES ARE EQUAL?

If the number 5 is compared to the number 7, then it is clear that 7 is greater. However, what if you compare  $x$  with 7? In this case,  $x$  could be smaller, larger, or equal to 7.



Examine the Expression Comparison Mat below.

- If the left expression is smaller than the right expression, what does that tell you about the value of  $x$ ?
- If the left expression is greater than the right expression, what does that tell you about the value of  $x$ ?

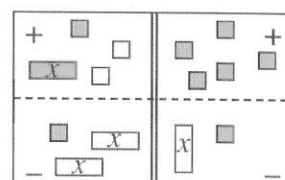


- What if the left expression is equal to the right expression? What does  $x$  have to be for the two expressions to be equal?

A-79. SOLVING FOR  $x$

Later in this course, you will learn more about situations like parts (a) and (b) in the preceding problem, called “inequalities.” For now, to learn more about  $x$ , assume that the left expression and the right expression are equal. The two expressions will be brought together on one mat to create an **Equation Mat**, as shown in the figure below. The double line down the center of an Equation Mat represents the word “equals.” It is a wall that separates the left side of an equation from the right side.

- Obtain the “Equation Mat” resource page from your teacher. Using algebra tiles, build the equation represented by the Equation Mat at right. Simplify as much as possible and then solve for  $x$ . Be sure to record your work.



- Using algebra tiles, build the equation  $2x - 5 = -1 + 5x + 2$  by placing  $2x - 5$  on the left side and  $-1 + 5x + 2$  on the right side. Then use your simplification skills to simplify this equation as much as possible so that  $x$  is alone on one side of the equation. Use the fact that both sides are equal to solve for  $x$ . Record your work.

A-80. Now apply this new solving skill by building, simplifying, and solving each equation below for  $x$ . Record your work.

a.  $3x - 7 = 2$

b.  $1 + 2x - x = x - 5 + x$

c.  $3 - 2x = 2x - 5$


d.  $3 + 2x - (x + 1) = 3x - 6$

e.  $-(x + 3 - x) = 2x - 7$

f.  $-4 + 2x + 2 = x + 1 + x$

A-81. Verify your solutions to parts (b) through (d) of problem A-80 are correct by “showing the check.” That is, substitute your answer into the original equation to show that you have a true statement.

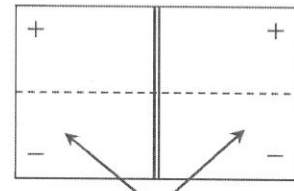
MATH NOTES



## METHODS AND MEANINGS

Using an Equation Mat

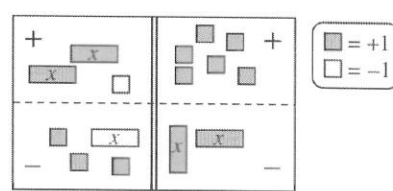
An Equation Mat can help you visually represent an equation with algebra tiles.



The double line represents the “equals” sign (=).

For each side of the equation, there is a positive and a negative region.

For example, the equation  $2x - 1 - (-x + 3) = 6 - 2x$  can be represented by the Equation Mat at right. (Note that there are other possible ways to represent this equation correctly on the Equation Mat.)





A-82. WHICH IS GREATER?

For each Expression Comparison Mat below, simplify and determine which side is greater.



a.

Left	Right
<div style="display: flex; justify-content: space-around;"> <div style="border: 1px solid black; padding: 5px; width: 40%;"> <math>+</math>  <div style="display: flex; justify-content: space-around;"> <div style="border: 1px solid black; width: 15px; height: 15px; background-color: gray;"></div> <div style="border: 1px solid black; width: 15px; height: 15px; background-color: white;"></div> <div style="border: 1px solid black; width: 15px; height: 15px; background-color: gray;"></div> <div style="border: 1px solid black; width: 15px; height: 15px; background-color: white;"></div> </div> </div> <div style="border: 1px solid black; padding: 5px; width: 40%;"> <math>+</math>  <div style="display: flex; justify-content: space-around;"> <div style="border: 1px solid black; width: 15px; height: 15px; background-color: gray;"></div> <div style="border: 1px solid black; width: 15px; height: 15px; background-color: white;"></div> <div style="border: 1px solid black; width: 15px; height: 15px; background-color: gray;"></div> <div style="border: 1px solid black; width: 15px; height: 15px; background-color: white;"></div> </div> </div> </div>	<div style="display: flex; justify-content: space-around;"> <div style="border: 1px solid black; padding: 5px; width: 40%;"> <math>+</math>  <div style="display: flex; justify-content: space-around;"> <div style="border: 1px solid black; width: 15px; height: 15px; background-color: gray;"></div> <div style="border: 1px solid black; width: 15px; height: 15px; background-color: white;"></div> <div style="border: 1px solid black; width: 15px; height: 15px; background-color: gray;"></div> <div style="border: 1px solid black; width: 15px; height: 15px; background-color: white;"></div> </div> </div> <div style="border: 1px solid black; padding: 5px; width: 40%;"> <math>+</math>  <div style="display: flex; justify-content: space-around;"> <div style="border: 1px solid black; width: 15px; height: 15px; background-color: gray;"></div> <div style="border: 1px solid black; width: 15px; height: 15px; background-color: white;"></div> <div style="border: 1px solid black; width: 15px; height: 15px; background-color: gray;"></div> <div style="border: 1px solid black; width: 15px; height: 15px; background-color: white;"></div> </div> </div> </div>
Which is greater?	

b.

Left	Right
<div style="display: flex; justify-content: space-around;"> <div style="border: 1px solid black; padding: 5px; width: 40%;"> <math>+</math>  <div style="display: flex; justify-content: space-around;"> <div style="border: 1px solid black; width: 15px; height: 15px; background-color: gray;"></div> <div style="border: 1px solid black; width: 15px; height: 15px; background-color: white;"></div> <div style="border: 1px solid black; width: 15px; height: 15px; background-color: gray;"></div> <div style="border: 1px solid black; width: 15px; height: 15px; background-color: white;"></div> </div> </div> <div style="border: 1px solid black; padding: 5px; width: 40%;"> <math>+</math>  <div style="display: flex; justify-content: space-around;"> <div style="border: 1px solid black; width: 15px; height: 15px; background-color: gray;"></div> <div style="border: 1px solid black; width: 15px; height: 15px; background-color: white;"></div> <div style="border: 1px solid black; width: 15px; height: 15px; background-color: gray;"></div> <div style="border: 1px solid black; width: 15px; height: 15px; background-color: white;"></div> </div> </div> </div>	<div style="display: flex; justify-content: space-around;"> <div style="border: 1px solid black; padding: 5px; width: 40%;"> <math>+</math>  <div style="display: flex; justify-content: space-around;"> <div style="border: 1px solid black; width: 15px; height: 15px; background-color: gray;"></div> <div style="border: 1px solid black; width: 15px; height: 15px; background-color: white;"></div> <div style="border: 1px solid black; width: 15px; height: 15px; background-color: gray;"></div> <div style="border: 1px solid black; width: 15px; height: 15px; background-color: white;"></div> </div> </div> <div style="border: 1px solid black; padding: 5px; width: 40%;"> <math>+</math>  <div style="display: flex; justify-content: space-around;"> <div style="border: 1px solid black; width: 15px; height: 15px; background-color: gray;"></div> <div style="border: 1px solid black; width: 15px; height: 15px; background-color: white;"></div> <div style="border: 1px solid black; width: 15px; height: 15px; background-color: gray;"></div> <div style="border: 1px solid black; width: 15px; height: 15px; background-color: white;"></div> </div> </div> </div>
Which is greater?	

- A-83. Solve this problem by writing an equation using only one variable. Then write your answer with a complete sentence.

Mairé is thinking of two numbers. The first number is 14 less than the second number. When she adds them, she gets 40. Help her younger sister, Enya, figure out the numbers.

- A-84. Simplify each expression below as much as possible.

- a.  $3y - y + 5x + 3 - 7x$       b.  $-1 - (-5x) - 2x + 2x^2 + 7$   
 c.  $6x + 2 - 1 - 4x - 3 - 2x + 2$       d.  $\frac{2}{3}x - 3y + \frac{1}{3}x + 2y$

- A-85. Plot the points  $(0, 0)$ ,  $(3, 2)$ , and  $(6, 4)$  on graph paper. Then draw a line through the points. Name the coordinates of three more points on the same line.

- A-86. Mr. Dexter's teams earned the following scores on a quiz: 15, 20, 19, 20, 16, 20, 14, 18, and 17.

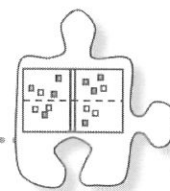
- a. What is the mean (average score)?  
 b. What is the median (middle score)?

- A-87. Simplify each expression.

- a.  $-\frac{2}{3} + \frac{4}{5}$       b.  $2 - \frac{3}{8}$       c.  $-\frac{3}{5} \cdot 10$       d.  $\frac{2}{3} \div -2$

# A.1.9 What is $x$ ?

## More Solving Equations



Today you will explore more equations on the Equation Mat and will examine all of the tools you have developed so far to solve for  $x$ . While you are working on these problems, be prepared to answer the following questions:

How can you simplify?

Can you get the variable alone?

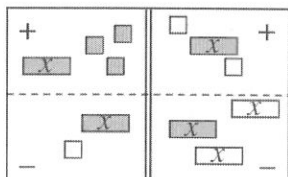
Is there more than one way to simplify?

Is there always a solution?

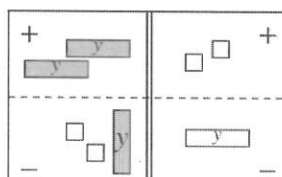
- A-88. On your paper, write the equation represented in each diagram below. For each equation, simplify as much as possible and then solve for  $x$  or  $y$ . Be sure to record your work on your paper.



a.



b.

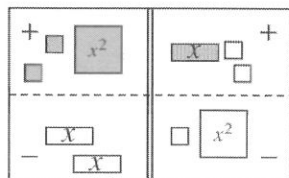


- A-89. IS THERE A SOLUTION?

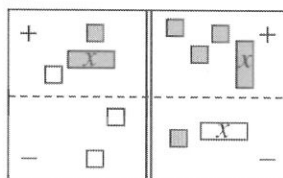
While solving homework last night, Richie came across three homework questions that he thinks have no solution. Build each equation below and determine if it has a solution for  $x$ . If it has a solution, find it. If it does not have a solution, explain why not.



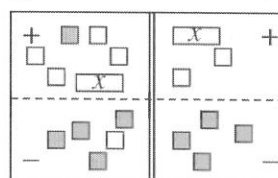
a.



b.



c.



- A-90. Continue to develop your equation-solving strategies by solving each equation below (if possible). Remember to build each equation, simplify as much as possible, and solve for  $x$  or  $y$ . There are often multiple ways to solve equations, so remember to justify that each step is “legal.” If you cannot solve for  $x$ , explain why not. Be sure to record your work.

a.  $-x + 2 = 4$


b.  $4x - 2 + x = 2x + 8 + 3x$

c.  $4y - 9 + y = 6$

d.  $9 - (2 - 3y) = 6 + 2y - (5 + y)$

- A-91. In your Learning Log, make a list of all the legal algebra tile moves. Then for each one, write its corresponding algebraic explanation. Title this entry “Using Algebra Tiles to Solve Equations” and include today’s date.





MATH NOTES

## METHODS AND MEANINGS

### Checking a Solution

To check a solution to an equation, substitute the solution into the equation and verify that it makes the two sides of the equation equal.

For example, to verify that  $x = 10$  is a solution to the equation  $3(x - 5) = 15$ , substitute 10 into the equation for  $x$  and then verify that the two sides of the equation are equal.

As shown at right,  $x = 10$  is a solution to the equation  $3(x - 5) = 15$ .

What happens if your answer is incorrect? To investigate this, test any solution that is not correct. For example, try substituting  $x = 2$  into the same equation. The result shows that  $x = 2$  is not a solution to this equation.

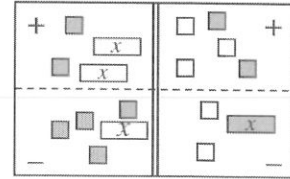
$$\begin{aligned}
 3(10 - 5) &\stackrel{?}{=} 15 \\
 3(5) &\stackrel{?}{=} 15 \\
 15 &= 15 \quad \checkmark \text{ Correct!}
 \end{aligned}$$
  

$$\begin{aligned}
 3(2 - 5) &\stackrel{?}{=} 15 \\
 3(-3) &\stackrel{?}{=} 15 \\
 -9 &\neq 15 \quad \times \text{ Not true, so } x = 2 \text{ is not a solution.}
 \end{aligned}$$



**Review & Preview**

- A-92. Translate the Equation Mat at right into an equation. Remember that the double line represents “equals.”

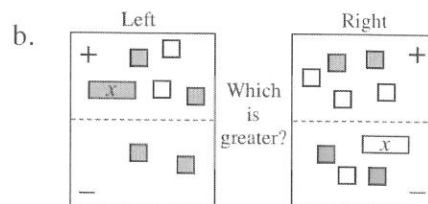
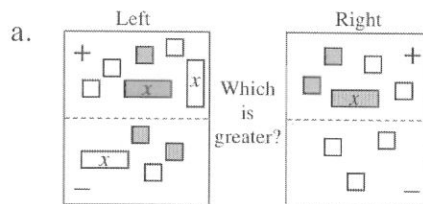


- A-93. Ling wants to save \$87 for tickets to a rock concert. If she has \$23 now and will save \$4 per week, how long will it take her to get enough money to buy the tickets?
- A-94. On graph paper, plot the points (0, 0), (−2, 1), and (2, −1). Then draw a line through them. Name the coordinates of three more points on the same line that have integer coordinates.
- A-95. Solve each equation and then show the check to prove that your answer is correct.

a.  $2 - 4x = 8 - x$

b.  $x + 4 - 3x = 4 - (x - 1)$

- A-96. Use legal simplification moves to determine which side of the Expression Comparison Mat is greater.



- A-97. Evaluate the expressions below for the given values.

a.  $6m + 2n^2$  for  $m = 7$  and  $n = 3$

b.  $\frac{5x}{3} - 2$  for  $x = -18$

c.  $(6x)^2 - \frac{x}{5}$  for  $x = 10$

d.  $(k - 3)(k + 2)$  for  $k = 1$

## Appendix Closure What have I learned?

### Reflection and Synthesis

The activities below offer you a chance to reflect about what you have learned during this chapter. As you work, look for concepts that you feel very comfortable with, ideas that you would like to learn more about, and topics you need more help with. Look for connections between ideas as well as connections with material you learned previously.



#### ① TEAM BRAINSTORM

What have you studied in this chapter? What ideas were important in what you learned? With your team, brainstorm a list. Be as detailed as you can. To help get you started, a list of Learning Log entries and Math Notes boxes are below.

What topics, ideas, and words that you learned *before* this chapter are connected to the new ideas in this chapter? Again, be as detailed as you can.

How long can you make your list? Challenge yourselves. Be prepared to share your team's ideas with the class.

#### Learning Log Entries

- Lesson A.1.1 – Variables
- Lesson A.1.2 – Finding Perimeter and Combining Like Terms
- Lesson A.1.3 – Meanings of Minus
- Lesson A.1.3 – Representing expressions on an Expression Mat
- Lesson A.1.4 – Using Zeros to Simplify
- Lesson A.1.5 – Legal Moves for Simplifying and Comparing Expressions
- Lesson A.1.9 – Solutions of an Equation



#### Math Notes

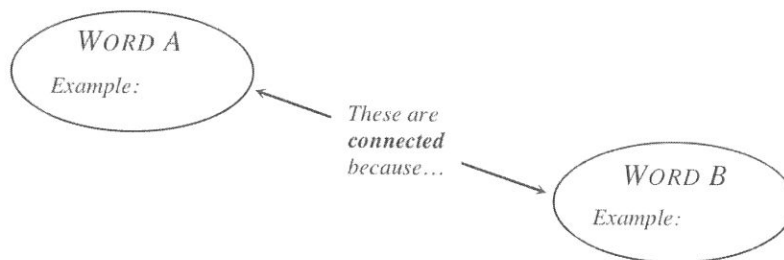
- Lesson A.1.1 – Non-Commensurate
- Lesson A.1.2 – Expression and Terms
- Lesson A.1.3 – Order of Operations
- Lesson A.1.4 – Evaluating Expressions and Equations
- Lesson A.1.5 – Combining Like Terms
- Lesson A.1.6 – Simplifying an Expression
- Lesson A.1.8 – Using an Equation Mat
- Lesson A.1.9 – Checking a Solution

## ② MAKING CONNECTIONS

Below is a list of the vocabulary used in this chapter. Make sure that you are familiar with all of these words and know what they mean. Refer to the glossary or index for any words that you do not yet understand.

combining like terms	algebra tiles	area
equal	Equation Mat	equation
variable	evaluate	expression
zero	Expression Mat	greater
minus	negative	opposite
order of operations	simplify	solution
solve	term	

Make a concept map showing all of the connections you can find among the key words and ideas listed above. To show a connection between two words, draw a line between them and explain the connection, as shown in the model below. A word can be connected to any other word as long as you can justify the connection. For each key word or idea, provide an example or sketch that shows the idea.



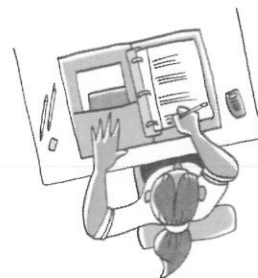
Your teacher may provide you with vocabulary cards to help you get started. If you use the cards to plan your concept map, be sure either to re-draw your concept map on your paper or to glue the vocabulary cards to a poster with all of the connections explained for others to see and understand.

While you are making your map, your team may think of related words or ideas that are not listed here. Be sure to include these ideas on your concept map.

③

## PORTFOLIO: EVIDENCE OF MATHEMATICAL PROFICIENCY

While simplifying the expressions shown in the Expression Comparison Mat below, the four members of a study team made the following statements. Which students justified their statements? And were the justifications convincing? Explain why or why not.

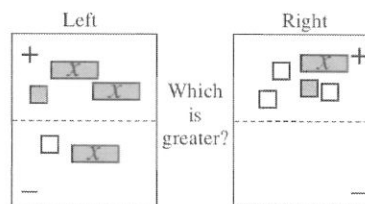


Rosalita: "I think we can take the positive unit tile and negative unit tile away from the left side because they make zero."

Anthony: "I think we can take an  $x$ -tile away from both sides."

Barry: "I don't think we can tell which side is greater because there are more  $x$ -tiles on the left side than on the right."

Deshawn: "I think we can remove a positive and negative unit tile from the "+" region on the right side because they are opposites, so they make zero."



Your teammate needs help understanding why  $-(-2x - 3) = 2x + 3$ . She thinks that  $-(-2x - 3) = 2x - 3$ . Justify why  $-(-2x - 3) = 2x + 3$  so that she is convinced.

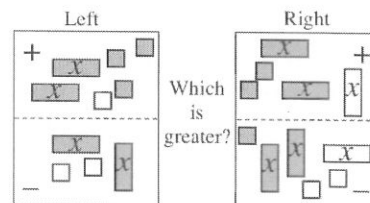
Obtain the Appendix Closure Resource Page: Simplifying/Solving Graphic Organizer from your teacher. Choose an equation that showcases what you have learned about solving an equation. Use the Resource Page to solve that equation. Make sure to show your tile diagram, the algebraic representation, and the step-by-step justification (legal tile moves). Then algebraically check your answer.

## WHAT HAVE I LEARNED?

CL A-98. Examine the Expression Mat at right.

- 
- Figure 1 shows a 2D grid of 10x10 cells. The top half (rows 1-5) is labeled 'X' and the bottom half (rows 6-10) is labeled 'Y'. A dashed line separates the two halves. Below the grid, a legend indicates that a shaded square represents +1 and an unshaded square represents -1.

CL A-100. Write expressions for each side of the Expression Comparison Mat. Use “legal” moves to simplify and determine which is greater.



- CL A-101. Solve the following problem by writing an equation using only one variable. Write your answer with a complete sentence.

Ralph and Alphonse are shooting marbles. Ralph has five more marbles than Alphonse, and they have a total of 73 marbles. How many marbles does each of them have?

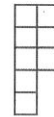
- CL A-102. Simplify each expression with or without algebra tiles. Record your steps.

a.  $3 + 7x - (2 + 9x)$

b.  $6 - (3x - 4) + 7x - 11$

- CL A-103. Copy the pattern below onto graph paper. Draw Figures 1 and 5 on your paper.

- a. How many tiles are in each figure?



- b. How is the pattern changing?

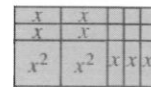
Figure 2

Figure 3

Figure 4

- c. How many tiles would Figure 6 have?

- CL A-104. Find the area and perimeter of the figure at right.



- CL A-105. Evaluate  $6x - (3y + 7) - xy$  when  $x = 5$  and  $y = 3$ .

- CL A-106. Simplify the expression below by combining like terms:

$$3x^2 + 10 - y^2 + 4x - 8x^2 - 5y - 8 + y^2 + 3$$

- CL A-107. Solve this equation to find  $x$  and then show the check:

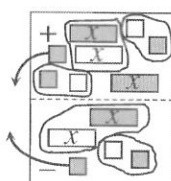
$$2 - (3x - 4) = 2x - 9$$


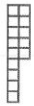
- CL A-108. Check your answers using the table at the end of the closure section. Which problems do you feel confident about? Which problems were hard? Use the table to make a list of topics you need help on and a list of topics you need to practice more.

## Answers and Support for Closure Activity #4

### What Have I Learned?

Note: MN = Math Note, LL = Learning Log

Problem	Solution	Need Help?	More Practice
CL A-98.	<div><div>b. <math>2x - x + 3 - 2 - (x - x + 2 - 1)</math></div><div>c. One possible answer:</div><div>d. <math>x</math></div></div> <div></div>	Lessons A.1.3 and A.1.4 MN: A.1.5 LL: A.1.3 and A.1.4	Problems A-27, A-30, A-37, A-39, A-40, A-43, A-45, and A-61
CL A-99.	112.5 minutes	Explanations and practice of topics from previous courses are available in the <i>Core Connections, Courses 1-3 Parent Guide with Extra Practice</i> , available free at <a href="http://www.cpm.org">www.cpm.org</a> .	Problems A-58, A-64, and A-76
CL A-100.	<div>Left: <math>-1 + 2x + 3 - (2x - 2) = 4</math></div> <div>Right: <math>2x + 2 - x - (2x - x - 2 + 1) = 3</math></div> <div>The left expression is greater than the right expression.</div>	Lesson A.1.6 MN: A.1.6 LL: A.1.5	Problems A-61, A-72, A-77, and A-82
CL A-101.	<div>Let <math>x</math> = Alphonse's number of marbles,</div> <div><math>x + (x + 5) = 73</math></div> <div>Ralph has 39 marbles, and Alphonse has 34 marbles.</div>	Explanations and practice of topics from previous courses are available in the <i>Core Connections, Courses 1-3 Parent Guide with Extra Practice</i> , available free at <a href="http://www.cpm.org">www.cpm.org</a> .	Problems A-32, A-71, A-83, and A-93

Problem	Solution	Need Help?	More Practice
CL A-102. a. $-2x+1$ b. $4x-1$		Lesson A.1.7	Problem A-84
CL A-103. a.	 Figure 1  Figure 5	Lesson 1.1.2	Problems CL 1-85 and A-73
	b. Each figure has three more tiles than the one before it. c. Figure 6 would have 17 tiles.		
CL A-104. Perimeter = $6x+10$ Area = $2x^2+7x+6$		Lesson A.1.1 and A.1.2	Problems A-21, A-46, and A-74
CL A-105. $6 \cdot 5 - (3 \cdot 3 + 7) - 5 \cdot 3 = -1$		MN: A.1.3 and A.1.4	Problems A-10, A-18, A-44, A-57, and A-97
CL A-106. $-5x^2+4x-5y+5$		Lessons A.1.1, A.1.3, A.1.2, and A.1.4 MN: A.1.5 LL: A.1.2, A.1.3, and A.1.4	Problems A-4, A-17, A-30, A-53, and A-84
CL A-107. $x=3$		Lessons A.1.8 and A.1.9 MN: A.1.9 LL: A.1.9	Problems A-80, A-90, and A-95





## Core Connections Algebra Checkpoint Materials

Notes to Students (and their Teachers)

Students master different skills at different speeds. No two students learn exactly the same way at the same time. At some point you will be expected to perform certain skills accurately. Most of the Checkpoint problems incorporate skills that you should have developed in previous courses. If you have not mastered these skills yet it does not mean that you will not be successful in this class. However, you may need to do some work outside of class to get caught up on them.

Starting in Chapter 1 and finishing in Chapter 11, there are 15 problems designed as Checkpoint problems. Each one is marked with an icon like the one above and numbered according to the chapter that it is in. After you do each of the Checkpoint problems, check your answers by referring to this section. If your answers are incorrect, you may need some extra practice to develop that skill. The practice sets are keyed to each of the Checkpoint problems in the textbook. Each has the topic clearly labeled, followed by the answers to the corresponding Checkpoint problem and then some completed examples. Next, the complete solution to the Checkpoint problem from the text is given, and there are more problems for you to practice with answers included.

Remember, looking is not the same as doing! You will never become good at any sport by just watching it, and in the same way, reading through the worked examples and understanding the steps is not the same as being able to do the problems yourself. How many of the extra practice problems do you need to try? That is really up to you. Remember that your goal is to be able to do similar problems on your own confidently and accurately. This is your responsibility. You should not expect your teacher to spend time in class going over the solutions to the Checkpoint problem sets. If you are not confident after reading the examples and trying the problems, you should get help outside of class time or talk to your teacher about working with a tutor.

Another source for help with the Checkpoint problems and other topics in *Core Connections Algebra* is the *Parent Guide with Extra Practice*. This resource is available for download free of charge at [www.cpm.org](http://www.cpm.org).

### Checkpoint Topics

1. Solving Linear Equations, Part 1 (Integer Coefficients)
2. Evaluating Expressions and the Order of Operations
3. Operations with Rational Numbers
4. Solving Linear Equations, Part 2 (Fractional Coefficients)
- 5A. Laws of Exponents and Scientific Notation
- 5B. Writing the Equation of a Line
- 6A. Rewriting Equations with More Than One Variable
- 6B. Multiplying Polynomials and Solving Equations with Parentheses
- 7A. Solving Problems by Writing Equations
- 7B. Solving Linear Systems of Equations
8. Interpreting Associations
9. Writing Exponential Equations from Situations
- 10A. The Exponential Web
- 10B. Factoring Polynomials
11. The Quadratic Web